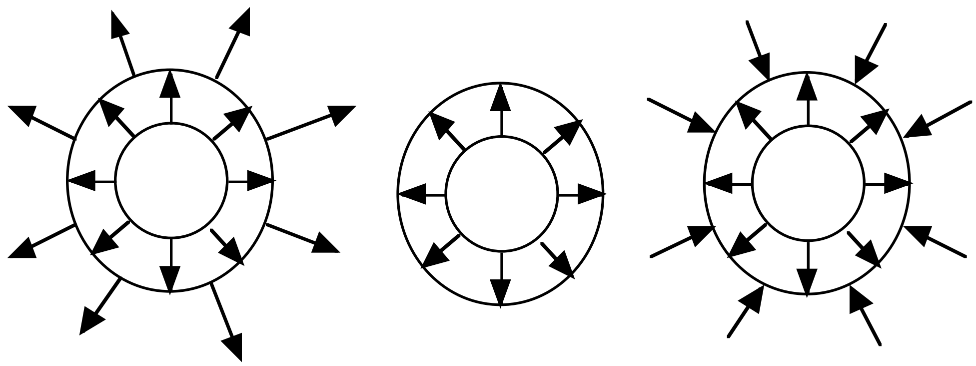
Chapter 22

22.2 Sketch qualitatively the electric field lines both between and outside two concentric conducting spherical shells when uniform positive charge q_1 is on the inner shell and a uniform negative charge $-q_2$ is on the outer. Consider the cases $q_1 > q_2$, $q_1 = q_2$, and $q_1 < q_2$.



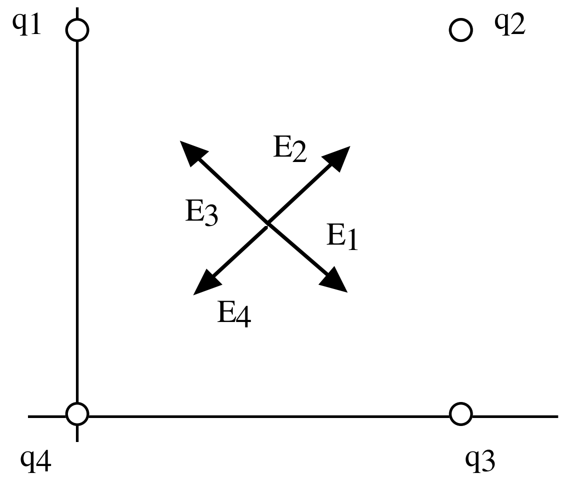
q1> q2 q1= q2 q1< q2

22.5 An atom of plutonium-239 has a nuclear radius of 6.64 fm and atomic number Z=94. Assuming that the positive charge is distributed uniformly within the nucleus, what are the magnitude and direction of the electric field at the surface of the nucleus due to the positive charge.

Outside a uniformly charged sphere, the field looks like that of a point charge at the center of the sphere.

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$
$$= \frac{94 \cdot 1.6\times10^{-19}C}{4\pi\varepsilon_0(6.64\times10^{-15}m)^2}$$
$$= 3.07\times10^{21}N/C!$$

22.9 In Fig 22-32, the four particles form a square of edge length $a = 5.00cm$ and have charges $q_1 = +10nC$, $q_2 = -20nC$, $q_3 = +20nC$, and $q_4 = -10nC$. In unit-vector notation, what net electric field do the particle produce at the square's center.



We can write out each field in unit vector notation and then add. Here all the q's are magnitudes since we have already included the directions by hand.

$$r = \frac{1}{2}\sqrt{a^2 + a^2} = \frac{\sqrt{2}}{2}a$$

$$\vec{E}_1 = \frac{q_1}{4\pi\varepsilon_0 \cdot a^2/2}\cos 45\,\hat{i} - \frac{q_1}{4\pi\varepsilon_0 \cdot a^2/2}\sin 45\,\hat{j} = \frac{q_1\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{i} - \frac{q_1\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{j}$$

$$\vec{E}_2 = \frac{q_2}{4\pi\varepsilon_0 \cdot a^2/2}\cos 45\,\hat{i} + \frac{q_2}{4\pi\varepsilon_0 \cdot a^2/2}\sin 45\,\hat{j} = \frac{q_2\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{i} + \frac{q_2\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{j}$$

$$\vec{E}_3 = -\frac{q_3}{4\pi\varepsilon_0 \cdot a^2/2}\cos 45\,\hat{i} + \frac{q_3}{4\pi\varepsilon_0 \cdot a^2/2}\sin 45\,\hat{j} = -\frac{q_3\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{i} + \frac{q_3\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{j}$$

$$\vec{E}_4 = -\frac{q_4}{4\pi\varepsilon_0 \cdot a^2/2}\cos 45\,\hat{i} - \frac{q_1}{4\pi\varepsilon_0 \cdot a^2/2}\sin 45\,\hat{j} = -\frac{q_4\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{i} - \frac{q_4\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{j}$$

$$E = \left(\frac{q_1\sqrt{2}}{4\pi\varepsilon_0 a^2} + \frac{q_2\sqrt{2}}{4\pi\varepsilon_0 a^2} - \frac{q_3\sqrt{2}}{4\pi\varepsilon_0 a^2} - \frac{q_4\sqrt{2}}{4\pi\varepsilon_0 a^2}\right)\hat{i}$$

$$+\left(-\frac{q_1\sqrt{2}}{4\pi\varepsilon_0 a^2} + \frac{q_2\sqrt{2}}{4\pi\varepsilon_0 a^2} + \frac{q_3\sqrt{2}}{4\pi\varepsilon_0 a^2} - \frac{q_4\sqrt{2}}{4\pi\varepsilon_0 a^2}\right)\hat{j}$$

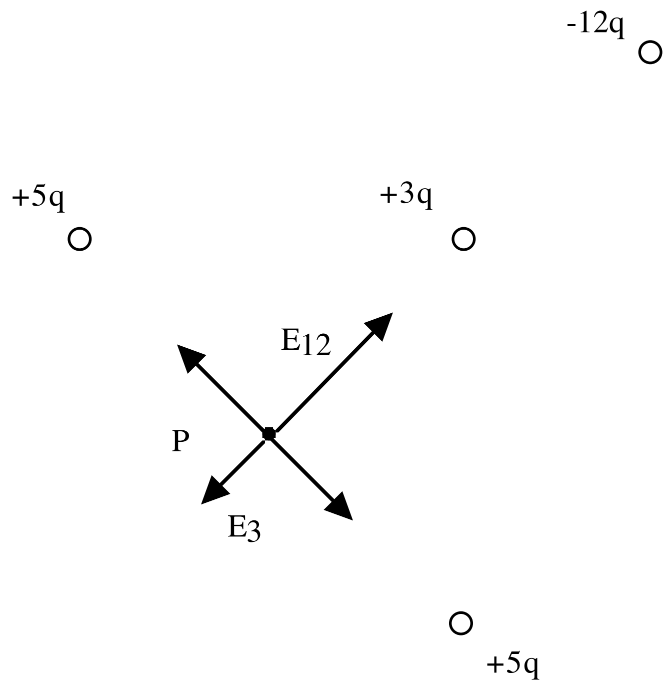
$$E = \frac{\sqrt{2}}{4\pi\varepsilon_0 a^2}(q_1 + q_2 - q_3 - q_4)\hat{i} + \frac{\sqrt{2}}{4\pi\varepsilon_0 a^2}(-q_1 + q_2 + q_3 - q_4)\hat{j}$$

$$= 0\hat{i} + 1.017 \times 10^5\,\hat{j}$$

22.10 In Fig. 22-33, what is the magnitude of the electric field at point P due to the four point charges shown?

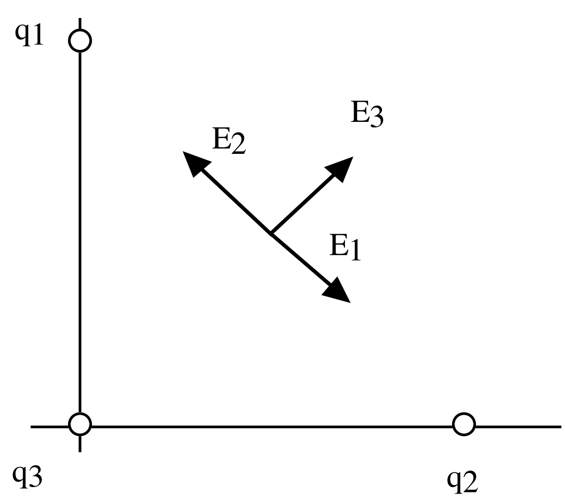
The fields due to the two +5q charges cancel exactly. We only need consider the fields due to the

+3q and -12q charges.



$$E = \frac{+3q}{4\pi\varepsilon_0 d^2} + \frac{-12q}{4\pi\varepsilon_0 (2d)^2}$$
$$= 0!$$

22.13 In Fig. 22-36, three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00\,\mu m$. What are the (a)magnitude and (b) direction of the net electric field at point P due to the particles.

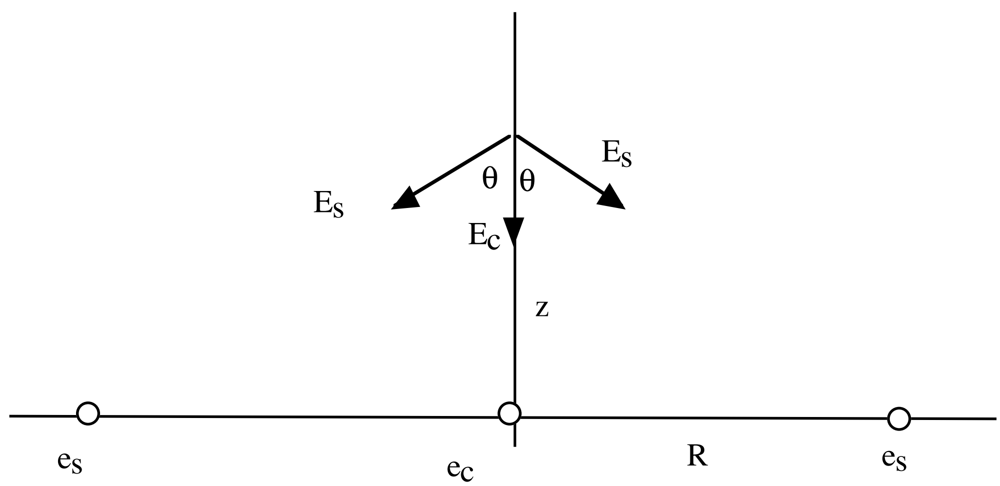


The fields due to charges 1 and 2 cancel exactly. This leaves us only to calculate the field due to 3.

$$r = \frac{1}{2}\sqrt{a^2 + a^2} = \frac{\sqrt{2}}{2}a$$

$$E_3 = \frac{q_3}{4\pi\varepsilon_0 \cdot a^2/2} = 159.8 N/C \ at \ 45°$$

22.15 Figure 22-38 shows a proton (pP on the central axis through a disk with a uniform charge density due to excess electrons . Three of those electrons are shown: electron e_c at the disk center and electrons e_s at opposite sides of the disk , at radius R from the center. The proton is initially a distance $z = R = 2.00cm$ from the disk. At that location, what are the magnitudes of (a) the electric field $\vec{E}_c$ due to electron e_c and (b) the net electric field $\vec{E}_{net,s}$ due to electrons e_s . The proton is then moved to $z = R/10.0$. What are the magnitudes of (c) $\vec{E}_c$ and (d) $\vec{E}_{net,s}$ at the proton's location. (e) From (a) and (c), we see that as the proton gets nearer to the disk, the magitude of $\vec{E}_c$ increases. Why does the magnitude of $\vec{E}_{net,s}$ decrease as we see from (b) and (d).



$$r_c = z$$

$$E_c = \frac{e}{4\pi\varepsilon_0 z^2}$$

$$r_s = \sqrt{R^2 + z^2}$$

$$E_{sz} = \frac{e}{4\pi\varepsilon_0 (R^2 + z^2)} \cdot \cos\theta$$

$$= \frac{e}{4\pi\varepsilon_0 (R^2 + z^2)} \cdot \frac{z}{\sqrt{R^2 + z^2}}$$

$$= \frac{ez}{4\pi\varepsilon_0 (R^2 + z^2)^{3/2}}$$

$$E_{snet} = 2E_{sz} = \frac{2ez}{4\pi\varepsilon_0 (R^2 + z^2)^{3/2}}$$

a) $E_c = \frac{e}{4\pi\varepsilon_0 z^2} = 3.59\times10^{-6}$

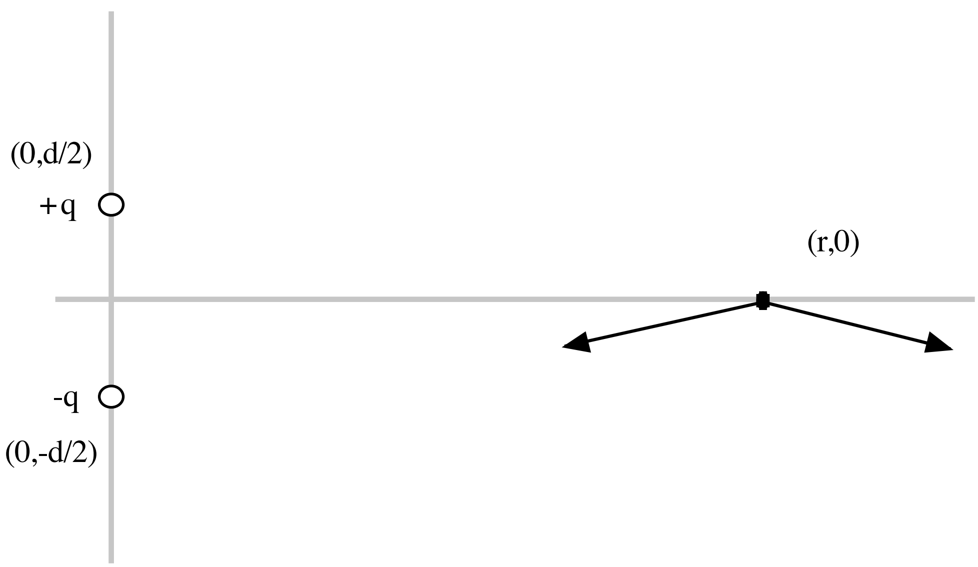
b) $E_{snet} = \frac{2ez}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} = 2.55\times10^{-6}$

c) $E_c = \frac{e}{4\pi\varepsilon_0 z^2} = 3.59\times10^{-4}$

d) $E_{snet} = \frac{2ez}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} = 7.087\times10^{-7}$

As the point gets closer to the disk, the z-component of the side electron fields gets smaller as the angle gets larger...

22.19. Find the magnitude and direction of the electric field at point P due to the electric dipole. You may assume that r>>d



We begin by writing the exact field.

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{q}{4\pi\varepsilon_0 r_+^2}\hat{r}_+ - \frac{q}{4\pi\varepsilon_0 r_-^2}\hat{r}_-$$

$$\vec{r}_+ = (r-0)\hat{i} + (0-\frac{d}{2})\hat{j} \qquad \vec{r}_- = (r-0)\hat{i} + (0-(-\frac{d}{2}))\hat{j}$$

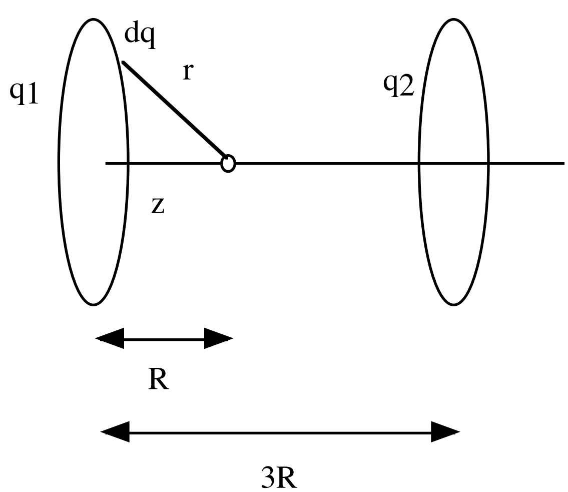
$$r_+ = \sqrt{r^2 + (\frac{d}{2})^2} \qquad r_- = \sqrt{r^2 + (\frac{d}{2})^2}$$

$$\hat{r}_+ = \frac{\vec{r}_+}{r_+} = \frac{r\hat{i} - \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}} \qquad \hat{r}_- = \frac{\vec{r}_-}{r_-} = \frac{r\hat{i} + \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}}$$

We can substitute in. In the last step, since d/2r is very small we ignore this term. Remember that the direction of p is in the +j direction in this problem (from - to + charge).

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$
$$= \frac{q}{4\pi\varepsilon_0 r_+^2}\hat{r}_+ - \frac{q}{4\pi\varepsilon_0 r_-^2}\hat{r}_-$$
$$= \frac{q}{4\pi\varepsilon_0 (r^2 + (\frac{d}{2})^2)} \cdot \frac{r\hat{i} - \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}} - \frac{q}{4\pi\varepsilon_0 (r^2 + (\frac{d}{2})^2)} \cdot \frac{r\hat{i} + \frac{d}{2}\hat{j}}{\sqrt{r^2 + (\frac{d}{2})^2}}$$
$$= \frac{-qd}{4\pi\varepsilon_0 (r^2 + (\frac{d}{2})^2)^{3/2}}\hat{j}$$
$$= \frac{-p}{4\pi\varepsilon_0 r^3 (1 + (\frac{d}{2r})^2)^{3/2}}\hat{j}$$
$$\approx \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$$

22.23 Figure 22-43 shows two parallel conducting rings with their central axes along a common line . Ring 1 has uniform charge q_1 and radius R; ring 2 has uniform charge q_2 and the same radius R. The rings are separated by a distance 3R. The net electric field at P on the common line at a distance R from ring 1 is zero. What is the ratio q_1 / q_2?



We begin by computing the field due to the ring on the left. We will assume that the point is as a position z. This will give us a general expression that we can use for either ring, plugging in the correct distance as appropriate. We begin by defining a charge per unit length.

$$\lambda_1 = \frac{q_1}{2\pi R}$$

Because of the symmetry, we only need to compute the field along the axis (called z).

$$dE_z = dE\cos\theta$$

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

$$r = \sqrt{R^2 + z^2}$$

$$dq = \lambda_1 R\, d\varphi$$

$$\cos\theta = \frac{z}{\sqrt{R^2 + z^2}}$$

$$E_z = \int_0^{2\pi} dE_z$$

$$E_z = \int_0^{2\pi} \frac{\lambda_1 R\, d\varphi}{4\pi\varepsilon_0 (R^2 + z^2)} \cdot \frac{z}{\sqrt{R^2 + z^2}}$$

$$= \int_0^{2\pi} \frac{\lambda_1 R z\, d\varphi}{4\pi\varepsilon_0 (R^2 + z^2)^{3/2}}$$

$$= \frac{\lambda_1 R z}{2\varepsilon_0 (R^2 + z^2)^{3/2}}$$

We can use this expression to compute the field for each loop, add them together and set the sum equal to zero field.

$$E_{1z} = \frac{\lambda_1 R z}{2\varepsilon_0 (R^2 + z^2)^{3/2}}$$

$$= \frac{\lambda_1 R^2}{2\varepsilon_0 (2R^2)^{3/2}}$$

$$E_{2z} = \frac{\lambda_2 R (2R)}{2\varepsilon_0 (R^2 + (2R)^2)^{3/2}}$$

$$= \frac{2\lambda_2 R^2}{2\varepsilon_0 (5R^2)^{3/2}}$$

$$0 = E_{1z} + E_{2z}$$

$$= \frac{\lambda_1 R^2}{2\varepsilon_0 (2R^2)^{3/2}} + \frac{2\lambda_2 R^2}{2\varepsilon_0 (5R^2)^{3/2}}$$

$$= \frac{\lambda_1}{(2)^{3/2}} + \frac{2\lambda_2}{(5)^{3/2}}$$

$$\frac{\lambda_1}{(2)^{3/2}} = \frac{2\lambda_1}{(5)^{3/2}}$$

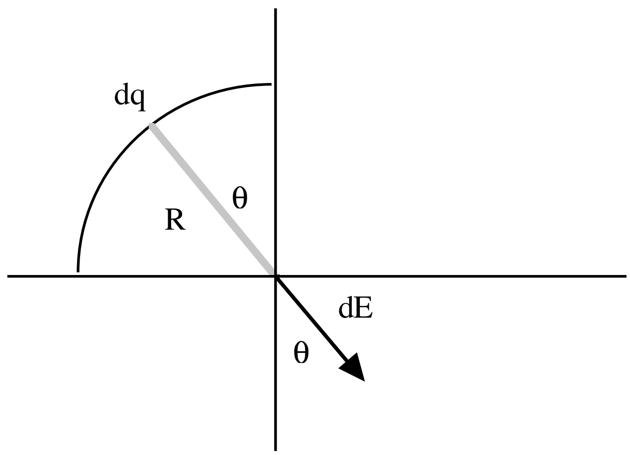
$$\frac{\lambda_1}{\lambda_2} = 2 \cdot (\frac{2}{5})^{3/2}$$

$$\frac{q_1}{q_2} = 2 \cdot (\frac{2}{5})^{3/2}$$

Since the rings have the same circumference, we can see that the ratio of the charge densities equals the ratio of the charges.

23.24 In Fig 22-45, a thing glass rod forms a semicircle of radius $r = 5.00cm$. Charge is uniformly distributed along the rod, with $+q = 4.50\,pC$ in the upper half and $-q = -4.50\,pC$ in the l lower half. What are the (a) magnitude and (b) direction relative to the positive direction of the x axis of the electric field E a P, the center of the circle.

Because of the symmetry in the problem, we can see that the net field will point downward. We also can see that the contribution from the bottom quarter circle is equal to the contribution from the top quarter circle. Because of this, we only need to compute the downward component due to the top quarter circle and multiply by 2.



We begin by defining a charge per unit length

$$\lambda = \frac{q}{\pi R/2}$$

We now find the component of interest and integrate...

$$dE_y = -dE\cos\theta$$

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

$$r = R$$

$$dq = \lambda R\,d\theta$$

$$E_y = \int_0^{\pi/2} dE_y$$

$$E_y = \int_0^{\pi/2} \frac{\lambda R\,d\theta}{4\pi\varepsilon_0 R^2}\cdot\cos\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta\,d\theta$$

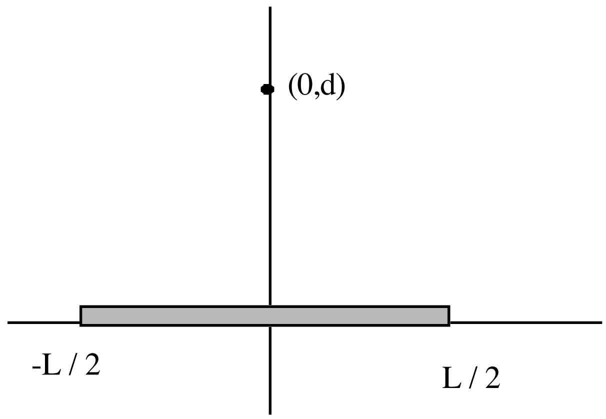
$$= \frac{\lambda}{4\pi\varepsilon_0 R}\cdot\frac{\pi}{2}$$

$$= \frac{\lambda}{8\varepsilon_0 R}$$

$$E_{y\,tot} = 2E_y = \frac{\lambda}{4\varepsilon_0 R}$$

The last factor of 2 is because there are two quarter rings.

22-32. In Fig. 22-51, positive charge $q = 7.81pC$ is spread uniformly along a thing non conducting rod of length $L = 14.5cm$. What are the (a) magnitude and (b) direction (relative to the x axis of the electric field produced at a distance $R = 6.00cm$ from the rod along its perpendicular bisector.



First we write dq for a little length of charge

$$dq = \lambda dx$$

We then write the $r, \vec{r}, \hat{r}$ for the charge dq.

$$r = \sqrt{(0-x)^2 + (d-0)^2}$$
$$\vec{r} = (0-x)\hat{i} + (d-0)\hat{j}$$
$$\hat{r} = \frac{\vec{r}}{r} = \frac{-x\,\hat{i} + d\,\hat{j}}{\sqrt{x^2 + d^2}}$$

We can then compute the x and y components of the Electric Field. You may need

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} = \int \frac{\lambda dx}{4\pi\varepsilon_0(x^2+d^2)} \frac{-x\hat{i}+d\hat{j}}{\sqrt{x^2+d^2}}$$

$$E_x = 0$$

$$E_y = \int \frac{\lambda d\, dx}{4\pi\varepsilon_0(x^2+d^2)^{3/2}} = \frac{\lambda d}{4\pi\varepsilon_0}\int \frac{dx}{(x^2+d^2)^{3/2}} = \frac{\lambda d}{4\pi\varepsilon_0}\cdot \frac{x}{d^2\sqrt{d^2+x^2}}\Bigg]_{-L/2}^{L/2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 d}\cdot\left[\frac{L/2}{\sqrt{d^2+L^2/4}} - \frac{-L/2}{\sqrt{d^2+L^2/4}}\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0 d}\cdot\frac{L}{\sqrt{d^2+L^2/4}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 d}\cdot\frac{L}{\sqrt{4d^2+L^2}}$$

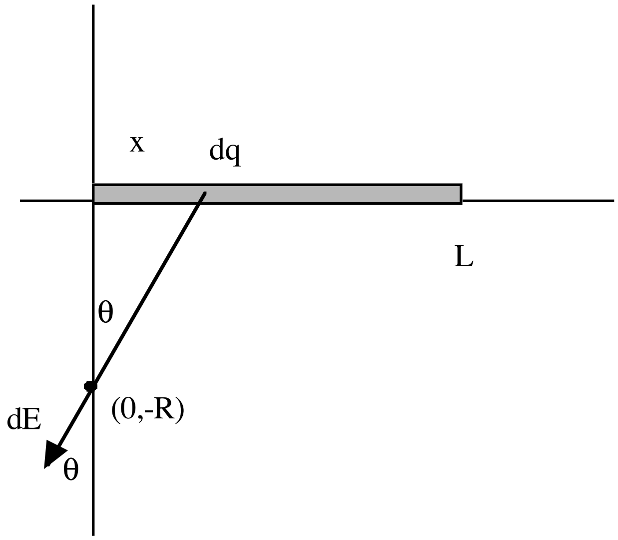
Now we can compute with numbers

$$\lambda = \frac{7.81\times10^{-12}C}{0.145m} = 5.386\times10^{-11}C\ /\ m$$

$$d = 0.06m$$

$$E_y == \frac{5.386\times10^{-11}C\ /\ m}{2\pi\varepsilon_0\cdot 0.06m}\cdot\frac{0.145m}{\sqrt{4(0.06m)^2+(0.145m)^2}} = 12.43N\ /\ C$$

22.33 In Fig 22-52, a "semi infinite" non conducting rod (that is, infinite in one direction only), has uniform linear charge density λ. Show that the electric field $\vec{E}_p$ at point P makes an angle of 45 degrees with the rod and this result is independent of the the distance R.



$$r = \sqrt{(0-x)^2 + (-R-0)^2}$$
$$\vec{r} = (0-x)\,\hat{i} + (-R-0)\,\hat{j}$$
$$\hat{r} = \frac{\vec{r}}{r} = \frac{-x\hat{i} - R\,\hat{j}}{\sqrt{x^2 + R^2}}$$

We can then compute the x and y components of the Electric Field. You may need

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

We can compute each integral and then take the limit as L goes to infinity. We find that in this limit, the components are identical for all R. This means that the ratio of these components is 1 and the angle is 45 degrees for all R.

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} = \int \frac{\lambda\, dx}{4\pi\varepsilon_0 (x^2+R^2)} \cdot \frac{-x\hat{i} - R\hat{j}}{\sqrt{x^2+R^2}}$$

$$E_x = \int \frac{\lambda\, dx}{4\pi\varepsilon_0 (x^2+R^2)} \cdot \frac{-x}{\sqrt{x^2+R^2}} = \int \frac{-x\lambda\, dx}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}} = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{R^2+x^2}}\Bigg]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \left(\frac{1}{\sqrt{R^2+L^2}} - \frac{1}{R}\right)$$

$$\lim_{:L\to\infty} E_x = -\frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{R}$$

$$E_y = \int \frac{-\lambda R\, dx}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}} = -\frac{\lambda R}{4\pi\varepsilon_0} \int \frac{dx}{(x^2+R^2)^{3/2}} = -\frac{\lambda R}{4\pi\varepsilon_0} \cdot \frac{x}{R^2\sqrt{R^2+x^2}}\Bigg]_0^L$$

$$= -\frac{\lambda}{4\pi\varepsilon_0 R} \cdot \left[\frac{L}{\sqrt{R^2+L^2}} - 0\right]$$

$$= -\frac{\lambda}{4\pi\varepsilon_0 R} \cdot \frac{L}{\sqrt{R^2+L^2}}$$

$$\lim_{L\to\infty} E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

22.35 At what distance along the central perpendicular axis of a uniformly charged plastic disk of radius 0.600m is the magnitude of the electric field equal to one half the magnitude of the field at the center of the disk.

$$\sigma = 5.3\times10^{-6} C/m^2$$
$$R = 0.6m$$
$$z = ?$$
$$E_z(z) = \frac{\sigma}{2\varepsilon_0}\cdot(1-\frac{z}{\sqrt{z^2+R^2}})$$
$$E_z(0) = \frac{\sigma}{2\varepsilon_0}\cdot(1-\frac{0}{\sqrt{0^2+R^2}})$$
$$= \frac{\sigma}{2\varepsilon_0}$$
$$\frac{1}{2}E_z(0) = E_z(z)$$
$$\frac{1}{2}\cdot\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}\cdot(1-\frac{z}{\sqrt{z^2+R^2}})$$
$$\frac{1}{2} = (1-\frac{z}{\sqrt{z^2+R^2}})$$
$$\frac{1}{2} = \frac{z}{\sqrt{z^2+R^2}}$$
$$z = \frac{R}{\sqrt{3}} = 0.3464m$$

22.45 Beams of high-speed protons can be produced in "guns" using electric fields to accelerate the protons. (a) What acceleration would a proton experience if the gun's electric field were $2.00\times10^4 N/C$? (W) What speed would the proton attain if the field accelerated the proton through a distance of $1.00cm$?

$$F = qE$$
$$ma = qE$$
$$a = \frac{q}{m}E = \frac{1.6\times10^{-19}}{1.67\times10^{-27}}\cdot 2.00\times10^4 N/C = 1.92\times10^{12} m/s^2$$
$$v_f^{\,2} = v_i^2 + 2a(x_f - x_i)$$
$$v_f^{\,2} = 0^2 + 2\cdot 1.92\times10^{12} m/s^2\cdot(0.01m) = 3.83\times10^{10} m^2/s^2$$
$$v_f = 1.957\times10^5 m/s$$