Chapter 21

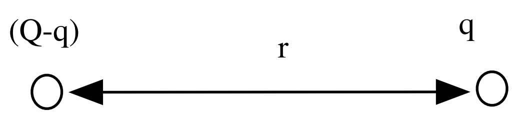
21.1 What must the distance between point charge $q_1 = 26\times10^{-6}C$ and point charge $q_2 = -47\times10^{-6}C$ for the electrostatic force between them to be 5.70N?

The magnitude of the force of attraction is given by Coulomb's law where we have taken the absolute value of the force and the charges.

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$
$$r = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 F}}$$
$$= \sqrt{\frac{26\times10^{-6}C\cdot 47\times10^{-6}C}{4\pi\varepsilon_0\cdot 5.7N}}$$
$$= 1.39m$$

21.5 Of the charge Q, on a tiny sphere, a portion q is to is to be transferred to a second nearby sphere. The spheres can be treated as particles. For what value of q/Q maximizes the magnitude F of the electrostatic force between the two spheres.

We begin with a drawing showing the charge on each sphere.

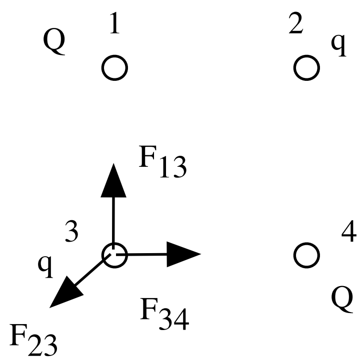


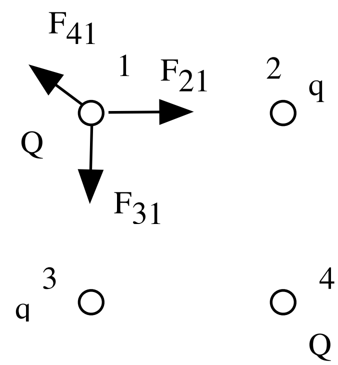
To find the maximum, we take the derivative and set it equal to zero.

$$F = \frac{q(Q-q)}{4\pi\varepsilon_0 r^2} = \frac{Qq-q^2}{4\pi\varepsilon_0 r^2}$$
$$\frac{dF}{dq} = 0 = \frac{Q-2q}{4\pi\varepsilon_0 r^2}$$
$$\frac{q}{Q} = \frac{1}{2}$$

21.8 In Fig 21-23, four particles for a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$
(a) What is Q/q if the net electrostatic force on particles 1 and 3 is zero. (b) Is there any value of q that makes the net electrostatic force on the each of the four particles zero? Explain.

We begin by drawing the forces on 1 and 3. We know that the diagonal force will be repulsive and we know that the magnitude of the remaining forces are identical (since the charges are identical). To get them to all balance, the q and Q must have opposite signs.





$$\vec{F}_{23} = -\frac{q \cdot q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45^\circ \hat{i} + -\frac{q \cdot q}{4\pi\varepsilon_0 \cdot 2a^2}\sin 45^\circ \hat{j}$$

$$\vec{F}_{34} = \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\hat{i}$$

$$\vec{F}_{13} = \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\hat{j}$$

$$\vec{F} = 0 = \vec{F}_{23} + \vec{F}_{34} + \vec{F}_{13}$$

$$0 = \left(-\frac{q \cdot q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45^\circ + \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)\hat{i} + \left(-\frac{q \cdot q}{4\pi\varepsilon_0 \cdot 2a^2}\sin 45^\circ + \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)\hat{j}$$

$$x - component$$

$$0 = \left(-\frac{q \cdot q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45^\circ + \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)$$

$$\frac{q \cdot q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45^\circ = \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}$$

$$\frac{Q}{q} = \frac{\cos 45^\circ}{2} = \frac{\sqrt{2}}{4}$$

$$y - component$$

$$\left(\frac{q \cdot q}{4\pi\varepsilon_0 \cdot 2a^2}\sin 45^\circ = \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)$$

$$\frac{Q}{q} = \frac{\sin 45^\circ}{2} = \frac{\sqrt{2}}{4}$$

We should now check the forces on charge 1.

$$\vec{F}_{41} = -\frac{Q \cdot Q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45°\,\hat{i} + \frac{Q \cdot Q}{4\pi\varepsilon_0 \cdot 2a^2}\sin 45°\,\hat{j}$$

$$\vec{F}_{21} = \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\hat{i}$$

$$\vec{F}_{31} = -\frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\hat{j}$$

$$\vec{F} = 0 = \vec{F}_{41} + \vec{F}_{21} + \vec{F}_{31}$$

$$0 = \left(-\frac{Q \cdot Q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45° + \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)\hat{i} + \left(\frac{Q \cdot Q}{4\pi\varepsilon_0 \cdot 2a^2}\sin 45° + -\frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)\hat{j}$$

$$x - component$$

$$0 = \left(-\frac{Q \cdot Q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45° + \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)$$

$$\frac{Q \cdot Q}{4\pi\varepsilon_0 \cdot 2a^2}\cos 45° = \frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}$$

$$\frac{Q}{q} = \frac{2}{\cos 45°} = \frac{4}{\sqrt{2}}$$

$$y - component$$

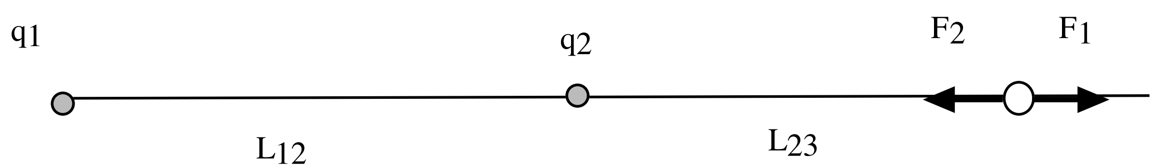
$$\left(\frac{Q \cdot Q}{4\pi\varepsilon_0 \cdot 2a^2}\sin 45° + -\frac{q \cdot Q}{4\pi\varepsilon_0 \cdot a^2}\right)$$

$$\frac{Q}{q} = \frac{2}{\sin 45°} = \frac{4}{\sqrt{2}}$$

It does not appear possible to find a ratio that will allow the force to be zero on 1 and 3 at the same time. It is also not possible to make the net force on every particle zero at the same time.

21.11 In Fig. 21-24, three charged particles lie on the x axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $L_{12} = L_{23}$, what is the ratio q_1 / q_2?

Again the forces must cancel. I will demonstrate another way to approach the problem Allow the q's to be either positive or negative. Let the charge at 3 be Q.



$$F_1 = \frac{q_1 Q}{4\pi\varepsilon_0 (L_{12} + L_{23})^2}$$

$$F_2 = \frac{q_2 Q}{4\pi\varepsilon_0 (L_{23})^2}$$

$$F_1 = F_2$$

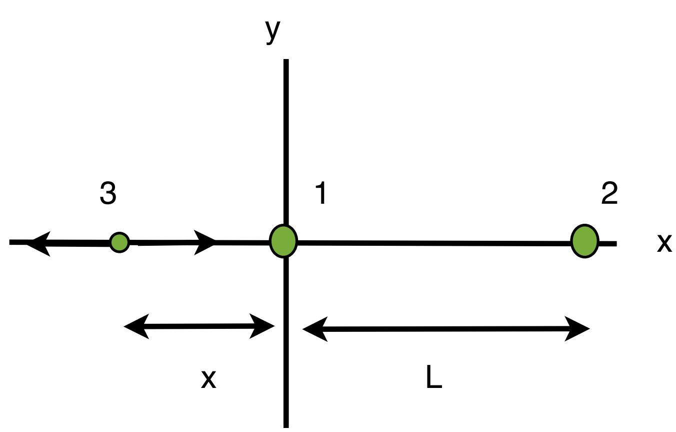
$$\frac{q_1 Q}{4\pi\varepsilon_0 (L_{12} + L_{23})^2} = \frac{q_2 Q}{4\pi\varepsilon_0 (L_{23})^2}$$

$$\frac{q_1}{(L_{12} + L_{23})^2} = \frac{q_2}{(L_{23})^2}$$

$$\frac{q_1}{q_2} = \frac{(L_{12} + L_{23})^2}{(L_{23})^2} = \frac{(2L)^2}{(L)^2} = 4$$

21.15 In Fig. 21-28, particle 1 of charge $+1\mu C$ and particle 2 of charge $-3\mu C$ are held at separation $L = 10.0cm$ on an x axis. If particle 3 of unknown charge q_3 is to located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) x and (b) y coordinates of particle 3.

Particle 3 must be on the x axis, to the left of 1 since it must be closer to 1 since particle 1 has a smaller charge. It must be in this region so that the net force can be zero. The sign of the charge of 3 determines which direction the forces due to 1 and 2 point, but their magnitudes must be equal and their directions are opposite.



We can write out the magnitudes and then set them equal

$$F_1 = \frac{q_1 q_3}{4\pi\varepsilon_0 x^2}$$

$$F_2 = \frac{q_2 q_3}{4\pi\varepsilon_0 (x+L)^2}$$

$$F_2 = F_1$$

$$\frac{q_2 q_3}{4\pi\varepsilon_0 (x+L)^2} = \frac{q_1 q_3}{4\pi\varepsilon_0 x^2}$$

$$\frac{q_2}{(x+L)^2} = \frac{q_1}{x^2}$$

$$q_2 x^2 = q_1 (x+L)^2$$

$$q_2 x^2 = q_1 (x^2 + 2xL + L^2)$$

$$0 = (q_1 - q_2)x^2 + 2q_1 Lx + q_1 L^2$$

$$x = \frac{-2q_1 L \pm \sqrt{4q_1^2 L^2 - 4(q_1 - q_2)q_1 L^2}}{2(q_1 - q_2)}$$

$$= \frac{-q_1 L \pm \sqrt{q_1^2 L^2 - (q_1 - q_2)q_1 L^2}}{2(q_1 - q_2)}$$

$$= 13.6\, cm$$

21.24 Two tiny spherical water drops with identical charges of $-1.00 \times 10^{-16} C$, have a center-to-center separation of 1.00 cm. (a) What is the magnitude of the electrostatic force acting between them? (b) How many excess electrons are on each drop, giving it its charge imbalance

$$F = \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{(1.0 \times 10^{-16})^2}{4\pi\varepsilon_0 (1.0 \times 10^{-2} m)^2} = 8.99 \times 10^{-19} N$$

$$\#\, electrons = \frac{1.0 \times 10^{-16} C}{1.6 \times 10^{-19} C / e} = 625$$

21.26 A current of 0.300 A through your chest can send your heart in fibrillation, ruining through normal rhythm of heart beat and disrupting the flow of blood (and thus oxygen) to your brain. If that current persists for 2.00 min., how many conduction electrons pass through your chest.

$$
\begin{aligned}
i &= \frac{\Delta q}{\Delta t} \\
\Delta q &= i\Delta t \\
&= 0.300A \cdot 120s \\
&= 36C
\end{aligned}
$$

$$\Delta q = \#\,electrons \cdot \frac{1.6\times10^{-19}C}{electron}$$

$$\#\,electrons = \frac{\Delta q}{\frac{1.6\times10^{-19}C}{electron}} = 2.25\times10^{20}$$

21.29 Earth's atmosphere is constantly bombarded by cosmic ray protons that originate somewhere in space. If the protons all passed through the atmosphere, each square meter of Earth's surface would intercept protons at the average rate of 1500 protons per second. What would be the corresponding electric current intercepted by the total surface area of the planet?

We take the rate times the charge per proton time the surface area of the planet.

$$
\begin{aligned}
i_{total} &= \frac{1500\,p}{s\cdot m^2} \cdot \frac{1.6\times10^{-19}C}{p} \cdot 4\pi\, r_e^{\,2} \\
&= 0.1224\,A
\end{aligned}
$$

21.36 Electrons and positrons are produced by the nuclear transformations of protons and neutrons known as beta decay. (a) If a proton transforms into a neutron, is an electron or a positron produced? (b) If a neutron transforms into a proton, is an electron or a positron produced.

(a) If a proton transforms into a neutron, charge conservation requires that a positron be produced.

(b) If a neutron transforms into a proton, charge conservation requires that an electron be produced.

21.37 Identify X in the following nuclear reactions (in the first, n represents neutron)

We must conserve both the total charge and the total number of protons + neutrons.

$$^{1}H + ^{9}Be \Rightarrow X + n$$

$$\begin{array}{llll} 1p & 4p & 5p & 0p \\ 0n & 5n & 4n & 1n \end{array} \qquad X = ^{9}B$$

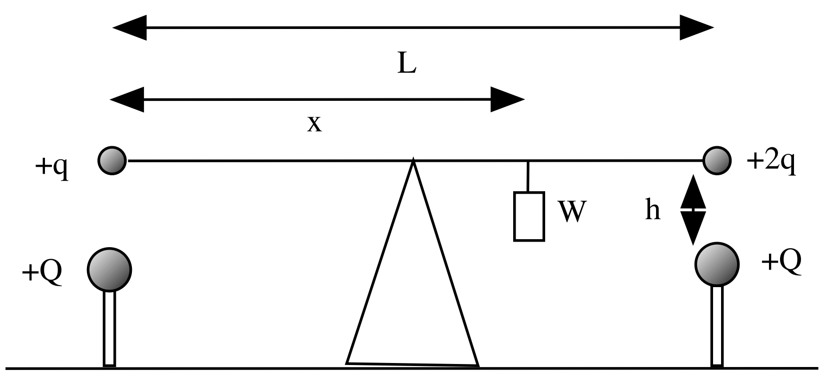
$$^{12}C + ^{1}H \Rightarrow X$$

$$\begin{array}{lll} 6p & 1p & 7p \\ 6n & 0n & 6n \end{array} \qquad X = ^{13}N$$

$$^{15}N + ^{1}H \Rightarrow ^{4}He + X$$

$$\begin{array}{llll} 7p & 1p & 2p & 6p \\ 8n & 0n & 2n & 6n \end{array} \qquad X = ^{12}C$$

21.44 Figure 21-39 shows a long, nonconducting, massless rod of length L, pivoted at its center and balanced with a block of weight W at a a distance x from the left end. At the left and right ends of the rod are attached small conducting sphere with positive charges q and 2q, respectively. A distance h directly beneath each of these spheres is a fixed sphere with a positive charge Q. (a) Find the distance x when the rod is horizontal and balanced. (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?



a) To proceed, we compute the torque about the central pivot. We will assume that torques that would cause clockwise acceleration are positive. Since the system is to remain at rest, the net torque must be zero.

$$0 = (x - \frac{L}{2}) \cdot W - \frac{L}{2} \cdot \frac{2qQ}{4\pi\varepsilon_0 h^2} + \frac{L}{2} \cdot \frac{qQ}{4\pi\varepsilon_0 h^2}$$

$$0 = x \cdot W - \frac{L}{2} \cdot W - \frac{L}{2} \cdot \frac{qQ}{4\pi\varepsilon_0 h^2}$$

$$x = \frac{L}{2} + \frac{L}{2W} \cdot \frac{qQ}{4\pi\varepsilon_0 h^2}$$

b) We can compute the net force and set it equal to zero to find h

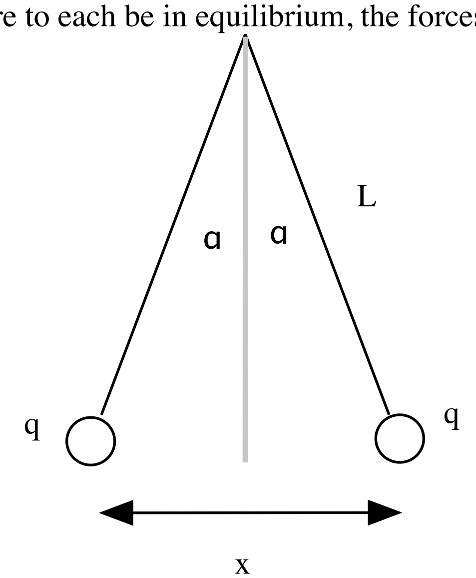
$$0 = \frac{2qQ}{4\pi\varepsilon_0 h^2} + \frac{qQ}{4\pi\varepsilon_0 h^2} - W$$
$$= \frac{3qQ}{4\pi\varepsilon_0 h^2} - W$$
$$\frac{3qQ}{4\pi\varepsilon_0 h^2} = W$$
$$h = \sqrt{\frac{3qQ}{4\pi\varepsilon_0 W}}$$

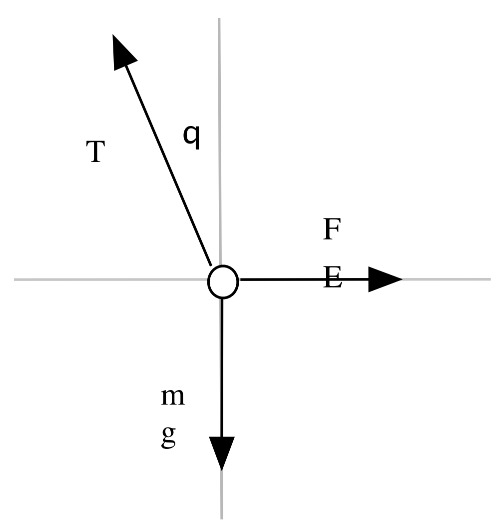
21.45 A Neutron consists of one "up" quark of charge +2e/3 and two "down" quarks each having charge -e/3. If we assume that the down quarks are $2.6 \times 10^{-15} m$ apart inside the neutron, what is the magnitude of the electrostatic force between them

$$F = \frac{q_d q_d}{4\pi \varepsilon_0 r^2} = 3.78N$$

21.54 In Fig. 21-42, two tiny conducting balls of identical mass m and identical charge q hang from nonconducting threads of length L. Assume that the angle is so small that the tan can be replaced with the sin. (a) Show that the equation displayed is necessary for equilibrium. (b) If L=120 cm, m=10g and x=5.0cm what is q?

We begin this problem by drawing a free-body diagram of the forces. Since we know that the balls are to each be in equilibrium, the forces must add to zero in each component direction.





$$\begin{aligned}&y\ direction\\&0 = T\cos\theta - mg\\&T = \frac{mg}{\cos\theta}\end{aligned}\qquad\qquad\begin{aligned}&x\ direction\\&0 = F_E - T\sin\theta\\&F_E = T\sin\theta\end{aligned}$$

$$\tan\theta \approx \sin\theta = \frac{x/2}{L}$$
$$F_E = T\sin\theta$$
$$qE = \frac{mg}{\cos\theta}\cdot\sin\theta$$
$$q\cdot\frac{q}{4\pi\varepsilon_0 x^2} = mg\tan\theta$$
$$\frac{q^2}{4\pi\varepsilon_0 x^2} = mg\cdot\frac{x/2}{L}$$
$$x^3 = \frac{q^2 L}{2\pi\varepsilon_0 mg}$$
$$x = (\frac{q^2 L}{2\pi\varepsilon_0 mg})^{1/3}$$

....as expected.

b) We invert this equation to find q.

$$x^3 = \frac{q^2 L}{2\pi\varepsilon_0 mg}$$
$$q^2 = \frac{2\pi\varepsilon_0 mgx^3}{L}$$
$$q = \sqrt{\frac{2\pi\varepsilon_0 mgx^3}{L}}$$
$$= 2.38\times10^{-8}C$$

21.65 In the radioactive decay of Eq. 21-13, a ^{238}U nucleus transforms to ^{234}Th and ejected ^{4}He. (these are nuclei, no atoms and thus electrons are not involved.). When the separation between ^{234}Th and ^{4}He is $9.0\times10^{-15}m$, what are the magnitudes of (a) the electrostatic force between them and (b) the acceleration of the ^{4}He particle.

$$F = \frac{q_{He}q_{Th}}{4\pi\varepsilon_0 r^2} = \frac{(2e)(90e)}{4\pi\varepsilon_0(9.0\times10^{-15}m)^2} = 511.5N$$
$$a_{He} = \frac{F}{m_{He}} = \frac{511.5N}{4\times(1.67\times10^{-27}kg)} = 7.66\times10^{+28}m/s^2$$